Efficient Allocation of Urgent Appointments?

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Introduction

Recent focus on achieving waiting time targets for outpatients and inpatients has led to increased awareness to the role which allocating urgent slots plays in influencing the waiting time of both the urgent and non-urgent patients.

The issue appears to be almost trivial. If the urgent waiting time is too high simply increase the relative allocation of urgent slots, either for a first outpatient appointment or for an urgent operation, and thereby the problem is solved. This strategy does however have the drawback that it can lengthen the waiting time for the remaining non-urgent patients.

The promised maximum two week wait for cancer referral has also led a perceived need for some of the urgent outpatient slots in a particular specialty to be reserved for cancer patients. In this instance we now have the potential for a very delicate balancing act to ensure that all classes of patient achieve the appropriate waiting time.

Most managers would agree that there must be a better way than trial and error to achieve these simultaneous objectives. Particularly so because guaranteed waiting time targets leave no room for the consequences of failure.

Fortunately a particular type of statistics called Poisson statistics allow us to develop solutions to these problems. Poisson statistics is the basis for what is called queuing theory. Queuing theory gives a description of the dynamic behaviour of any queue. To achieve this two fundamental pieces of information are required, namely, the arrival rate (e.g. number of cancer referrals per week) and the service rate (e.g. number of new appointment slots available each week to the arriving patients).

The apparent simplicity of allocating 2 slots per week to an expected 2 arrivals per week is in fact shattered by the fact that Poisson statistics tells us that outcomes other than the average are highly likely. In actual fact a Poisson distribution changes shape depending on the expected average and becomes highly skewed as the expected average decreases.

In order to make the following discussion of practical relevance we must first establish the level of urgent referrals received by most consultants. In the context of cancer referrals the largest weekly average is for combined upper and lower GI tract where a typical large general acute Trust would receive around 10 to 20 per week. This is spread over a number of consultants in both General Surgery and

Gastroenterology and hence the average per consultant will be less than 5 per week. In most instances the highest average of new referrals per consultant is usually less than 2 per week. The situation for the general category of urgent first outpatient appointments likewise gives low average arrival rates for all consultants with almost all consultants receiving fewer than 20 urgent requests per week and the majority receiving fewer than 10 per week. Urgent operations likewise occur at similarly low frequency.

Having established the boundaries we can now specifically investigate the effect of Poisson randomness on such small number events. This is summarised in Table One.

Table One: Likelihood of different outcomes given an expected average arrival rate per week.

Average urgent arrivals per week	% of weeks when there are no arrivals	% of weeks when there will be fewer arrivals than the average	% of weeks when there will be more arrivals than the average	% of weeks with average number of arrivals
1	37%	37%	26%	37%
2	14%	40%	32%	28%
3	5%	42%	35%	23%
4	2%	43%	37%	20%
5	1%	44%	38%	18%
6	0.2%	45%	39%	16%
7	0.1%	45%	40%	15%
8	0.03%	45%	41%	14%
9	0.01%	46%	42%	12%
10	0.005%	46%	44%	10%
20	0.0000002 %	47%	44%	9%

Several important points emerge from a consideration of Table One, namely:

- Even at an expected arrival rate of 20 per week it is possible (although with very low probability) to get one week in which there are no arrivals (second column)
- There are a higher proportion of weeks when there are less arrivals than the expected average (third column)
- Outcomes below the average occur more often than those above it (columns three and four)
- The average arrival rate does not occur with high likelihood (fifth column)

These observations lead us to a further uncomfortable question. How do we actually know the average expected arrival rate? The answer that comes back is usually that we count the referrals and take an average. Most managers who have studied statistics would point out that most textbooks indicate that it takes 30 or more measurements to establish an accurate average. This would imply that if we measure the arrivals for 30 weeks and take an average we should have an 'accurate' measure of the true average. In practice seasonal effects on referral rates and the occurrence of public holidays make anything less than a 52 week sample subject to considerable bias (1). However, Poisson statistics does have particular requirements and Table Two shows the accuracy obtained from one, two and three year sample periods.

Table Two: Effect of the number of measurements on the accuracy of the calculated average

True average arrivals per week	Maximum uncertainty in the calculated average given different sample sizes		
	52 weeks	104 weeks	156 weeks
1	0.48 – 1.50	0.72 – 1.29	0.74 – 1.23
10	8.1 – 11.65	8.91 – 10.93	9.22 – 10.81
20	18.13 – 21.88	18.73 – 21.45	18.97 –21.10

This table clearly shows that the accuracy of any attempts to estimate the average declines rapidly for average arrival rates below 20 per week, i.e. for all consultant clinics there will be high uncertainty regarding the average arrival rate. For example, at an average of 10 referrals per week there is a 19% uncertainty band in the calculated average using 52 weeks of data.

If we cannot even measure the average with accuracy how then can we allocate the correct number of slots?

Before returning to this question we need to investigate one further consequence of Poisson randomness and its impact on the efficient allocation of scarce resources. In this respect most consultants or managers would not wish to have empty clinic slots since this is clearly a waste of resource. To avoid this possibility we could theoretically set up a clinic with sufficient patients waiting at the start of the year to avoid the possibility of lower than average referrals leading to empty clinic slots toward the end of the year. This is explored in Table Three for various levels of urgent referral where the maximum and minimum referrals are at the 95% confidence intervals, i.e. higher and lower numbers of referrals will only occur on 5% of occasions..

Due to randomness in the arrival of urgent referrals we see that the minimum possible urgent wait to avoid wasting scarce resources is three weeks but that this could lead to a maximum wait of seven weeks due to higher than average arrival of referrals.

It would appear that Poisson randomness defeats all attempts to efficiently allocate scarce resources while simultaneously attempting to deliver a low waiting time, e.g. 2 weeks for cancer referral and 4 weeks for other types of urgent referral.

Table Three: Hypothetical clinic where number waiting at start of year is sufficient to avoid wasted clinic slots due to lower than average referrals

Average referral rate		Referrals actually received in year		Number Waiting	Waiting time (weeks)		
Per Year	Per week	Maximum	Minimum	Start of year	Start of year	Last day of year (Maximum)	Last day of year (Minimum)
1040	20	1105	975	65	3	7	0
936	18	997	875	61	3	7	0
832	16	890	774	58	4	7	0
728	14	782	674	54	4	8	0
624	12	674	574	50	4	8	0
520	10	566	474	46	5	9	0
416	8	457	375	41	5	10	0
312	6	348	276	36	6	12	0
260	5	293	227	33	7	13	0
208	4	237	179	29	7	15	0
156	3	181	131	25	8	17	0
104	2	124	84	20	10	20	0
52	1	66	38	14	14	28	0
26	0.5	36	16	10	20	40	0
10.4	0.2	17	4	6	32	65	0
5.2	0.1	10	1	4	42	90	0

Help & Advice

Contact the author for statistical advice on how to optimise the blend of outpatient appointments.